IIT-JEE – 2007 Paper – 2 Solutions

Physics

Part – I
Section – I
Straight Objective Type

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct

1. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-ray is

(A) $\lambda_0 = \frac{2mc\lambda^2}{h}$

(B) $\lambda_0 = \frac{2h}{mc}$

(C) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$

(D) $\lambda_0 = \lambda$

Sol. (A)

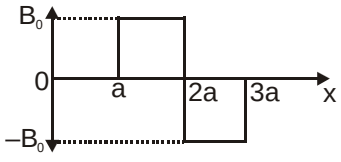
Cut – off wavelength corresponds to applied accelerating potential (V)

λ cut-off $= \frac{hc}{eV}$; $eV = \frac{1}{2}mv^2 = \frac{P^2}{2m}$ and $P = \frac{h}{\lambda}$

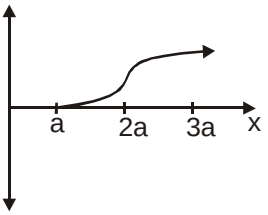
$\lambda = \lambda_0$ (cut-off) $= \frac{hc\lambda 2m\lambda^2}{h^2}$

$= \left(\frac{2mc}{h}\lambda\right)\lambda^2$

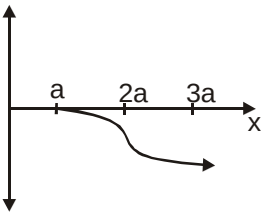
2. A magnetic field $B = B_0\hat{j}$ exist in the region $a < x < 2a$ and $B = -B_0\hat{j}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $v = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at x = a. The trajectory of the charge in this region can be like,



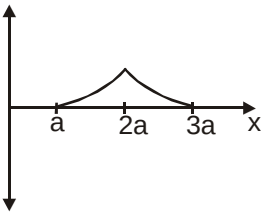
(A)



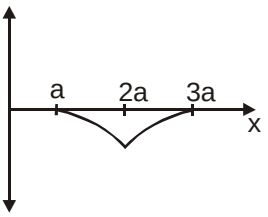
(B)



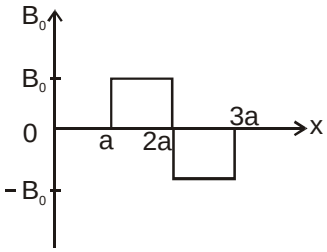
(C)



(D)



Sol. (A)



$B = B_0\hat{j}$; $a < x < 2a$

$= -B_0\hat{j}$, $2a < x < 3a$

$v = v_0\hat{i}$

$F = qv_0B_0\hat{k}$, $a < x < 2a$

$= qv_zB_0\hat{i} + qB_0v_0\hat{k}$, $2a < x < 3a$

$qv_zB_0\hat{i}$ is along x axis.

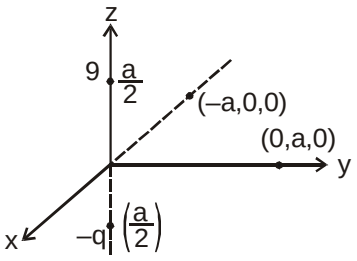
$-qB_0v_0\hat{k}$ is along –ve z-axis.

The charge at the time of entering $2a < x < 2a$ region, has some positive v_z . Just after it enters this region, a retarding force acts on it. Hence its z continues to increase till a certain time atleast. Hence (B), (C), (D) are eliminated. In the region $a < x < 2a$, z obviously is increasing.

3. Positive and negative point charges of equal magnitude are kept $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, -\frac{a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved form (–a, 0, 0) to (0, a, 0) is

(A) positive
(B) negative
(C) zero
(D) depends on the path connecting the initial and final positions

Sol. (C)

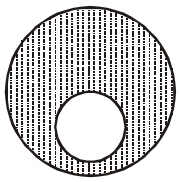


$V(0, a, 0) = 0$

$V(-a, 0, 0) = 0$

$\Rightarrow$ W.D $= 0$

4. A spherical portion has been removed form a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is

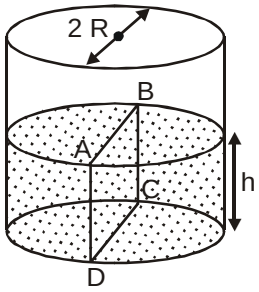


(A) zero everywhere
(B) non-zero and uniform
(C) non-uniform
(D) zero only at its center

Sol. (B)

This can be easily proved using superposition principle. Field would have been zero if cavity was concentric or the sphere was conducting.

5. Water is filled up to a height h in beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude



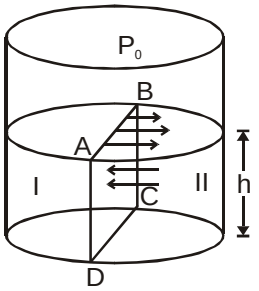
(A) $\left|2P_0 Rh\rho\pi R^2\rho gh\rho 2RT\right|$

(B) $\left|2P_0 Rh\rho\pi R\rho gh^2\rho 2RT\right|$

(C) $\left|P_0\rho\pi R^2\rho\pi R\rho gh^2\rho 2RT\right|$

(D) $\left|P_0\rho\pi R^2\rho\pi R\rho gh^2\rho 2RT\right|$

Sol. (B)

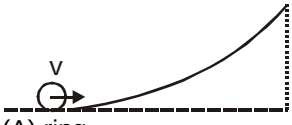


Force on water on side I
= Force due to water on side II (toward left)
– Force due to surface tension (towards right)

$= \left(P_0\pi\frac{\rho gh}{2}\right)2Rh - T\cdot 2R$

$\pi 2P_0Rh\rho\pi ghR^2 - 2TR$

6. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



(A) ring
(B) solid sphere
(C) hollow sphere
(D) Disc

Sol. (D)

$E_i = KE = \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\omega^2$

$V = \omega R$ (rolling)

$\Rightarrow E_i = \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\frac{V^2}{R^2}$

$E_f = mgh = mg\frac{3v^2}{4g}$

$E_i = E_f$ (W.D by friction is zero in pure rolling)

$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\frac{V^2}{R^2} = \frac{3mv^2}{4}$

or $\frac{I_{cm}}{2R^2} = \frac{m}{4}$

$\Rightarrow I_{cm} = \frac{mR^2}{2}$

$\Rightarrow$ object is a disc.

7. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $p(t) = A\left[\hat{i}\cos(kt) + \hat{j}\sin(kt)\right]$, where A and k are constants. The angle between the force and the momentum is

(A) 0°
(B) 30°
(C) 45°
(D) 90°

Sol. (D)

$P = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$

$F = \frac{dP}{dt} = A\left[-k\sin(kt)\hat{i} + k\cos(kt)\hat{j}\right]$

$F.P = A^2k\left[\sin(kt)\hat{i} - \cos(kt)\hat{j}\right]\left[-\cos(kt)\hat{i} + \sin(kt)\hat{j}\right]$

$= A^2k[0] = 0$

$\Rightarrow F \perp P$

8. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of $\pm$ 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of $\pm$ 0.01 mm. Take $g = 9.8\ m/s^2$ (exact). The Young's modulus obtained from the reading is

(A) $(2.0 \pm 0.3) \times 10^{11}\ N/m^2$

(B) $(2.0 \pm 0.2) \times 10^{11}\ N/m^2$

(C) $(2.0 \pm 0.1) \times 10^{11}\ N/m^2$

(D) $(2.0 \pm 0.05) \times 10^{11}\ N/m^2$

Sol. (B)

$$y = \frac{4Fl}{\pi d^2 l} \quad \ln y = \ln\left(\frac{4Fl}{\pi^2}\right) - 2\ln d - \ln l$$

$$\frac{\Delta y}{y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

$$= \frac{2 \times 0.01}{0.4} + \frac{0.05}{0.8}$$

$$= 0.05 + 0.06 = 0.1$$

Only option (B) gives $\frac{\Delta y}{y} = 0.1$

9. In the experiment to determine the speed of sound using a resonance column,
(A) prongs of the tuning fork are kept in a vertical plane
(B) prongs of the tuning fork are kept in a horizontal plane
(C) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(D) in one of the two resonance observed, the length of the resonating air column is close to half of the wavelength of sound in air

Sol. (A)

Section – II
Assertion – Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

10. **Statement - 1**
A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Because

Statement – 2
For every action there is an equal an opposite reaction.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (B)

S1 ? ?? Correct

S2 ? ?? Correct

The correct explanation to S1 is inertia.

11. **Statement - 1**
If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.

Because

Statement – 2
The linear momentum of an isolated system remains constant.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (B)

S1 ? ?? correct

S2 ? ?? correct.

$?^{ext}$? 0 , doesn't necessarily imply F^{ext} ? 0

12. **Statement - 1**
The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

Because

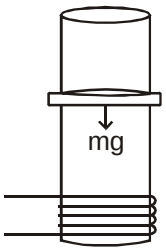
Statement – 2
The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (B)

S1 ? ?? Correct

S2 ? ?? Correct
But S2 is not a sufficient condition for S1 to be true. The reason being that the collision between molecules and that with the "walls" should be necessary elastic for S1 to be correct.

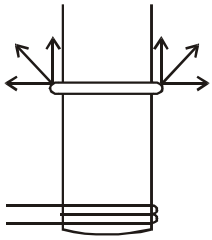
13. **Statement - 1**
A vertical iron rod has coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.



Because

Statement – 2
In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (A)



S1 ? ?? Correct

S2 ? ?? Correct
The magnetic field on the ring has two components. The horizontal component contributes to up upward vertical force to balance its weight.

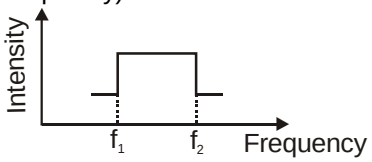
Section – III
Linked Comprehension Type

This section contains 2 paragraphs P14-16 and P 17-19. Based upon each paragraph, 3 multiple choice Questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **Only one** is correct.

P14-16 : Paragraph for Questions Nos. 14 to 16.

Two trains A and B are moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

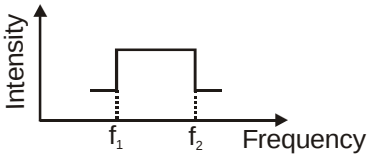
Assume that the sound of the whistle is composed of components varying in frequency from f_1 = 800 Hz to f_2 = 1120 Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

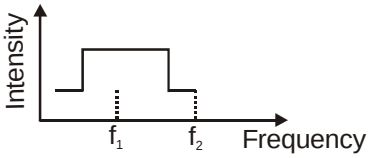


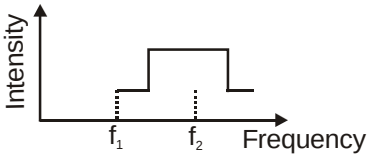
14. The speed of sound of the whistle is
(A) 340 m/s for passengers in A and 310 m/s for passengers in B
(B) 360 m/s for passengers in A and 310 m/s for passengers in B
(C) 310 m/s for passengers in A and 360 m/s for passengers in B
(D) 340 m/s for passengers in both the trains

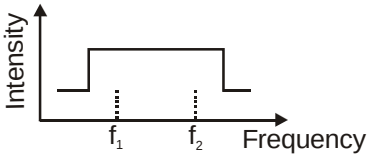
Sol. (D)
Speed of sound is not affected by motion of source on observer. It depends only on the medium and its state of motion w.r.t. the reference frame used.

15. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

(A)


(B)


(C)


(D)


Sol. (A)
For passengers in train A, the frequency heard is the same as the frequency emitted.

16. The spread of frequency as observed by the passengers in train B is
(A) 310 Hz
(B) 330 Hz
(C) 350 Hz
(D) 290 Hz

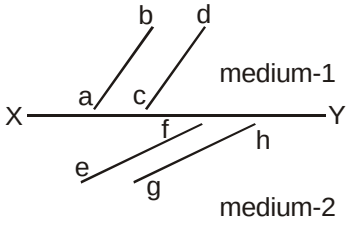
Sol. (A)

$$f' = f_0\left(\frac{340-30}{340-20}\right) = \frac{31}{32}f_0$$

$$\Delta f = f' - \frac{31}{32}f_0 = \frac{31}{32}\times 320 = 310\text{Hz}$$

P17-19 : Paragraph for Questions Nos. 17 to 19

The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of the light wave travelling in medium-1 and incident on XY. The lines of ef and gh represent wavefronts of the light wave in medum-2 after refraction.



17. Light travels as a
(A) parallel beam in each medium
(B) convergent beam in each medium
(C) divergent beam in each medium
(D) divergent beam in one medium and convergent beam in the other medium

Sol. (A)

Light rays are drawn perpendicular to the wave fronts. Since the wave fronts are parallel, the light rays are also parallel.

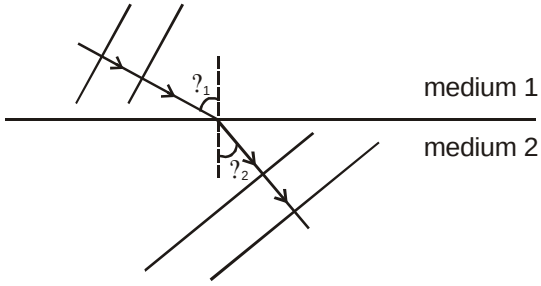
18. The phases of the light wave at c, d, e and are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c, \neq \phi_f$.
(A) ϕ_c cannot be equal to ϕ_d
(B) ϕ_d can be equal to ϕ_e
(C) $\left|\phi_d - \phi_f\right|$ is equal to $\left|\phi_c - \phi_e\right|$
(D) $\left|\phi_d - \phi_c\right|$ is not equal to $\left|\phi_f - \phi_e\right|$

Sol. (C)

Points on a wave front have the same phase.
$\therefore \phi_c = \phi_d$
and $\phi_e = \phi_f$
$\therefore \left|\phi_d - \phi_f\right| = \left|\phi_c - \phi_e\right|$

19. Speed of light is
(A) the same in medium-1 and medium-2
(B) larger in medium-1 than in medium-2
(C) larger in medium-2 than in midium-1
(D different at b and d

Sol. (B)



$n_1 \sin ?_1 ? n_2 \sin ?_2$

Since $?_1 ? ?_2$

$n_1 ? n_2$

Section – IV
Matrix-Match Type

This section contains 3 questions . Each question contains statement given in two columns which have to be matched. Statements (A, B, C, D) in **column I** have to be matched with statements (p, q, r, s) in **column II**. The answers to these questions have to be appropriately bubbled as illustrated in the following examples

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as
follows

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

20. **Column I** describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	The object moves on the x-axis under a conservative force in such a way that its "speed" and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	**(p)**	The object executes a simple harmonic motion.
(B)	The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	**(q)**	The object does not change its direction.
(C)	The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant accelerations a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	**(r)**	The kinetic energy of the object keeps on decreasing.
(D)	The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is radius of the earth. Neglect forces from object other than the earth.	**(S)**	The object can change its direction only once.

Sol. (A)

Suppose $x = x_0 \sin \omega t$

$v = \frac{dx}{dt} = x_0\omega\cos\omega t$

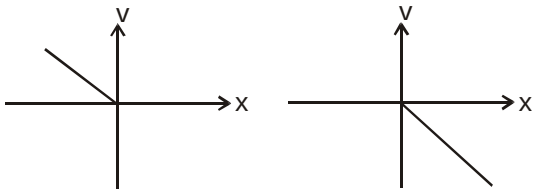
$= x_0\omega\sqrt{1-\sin^2\omega t}$

$= x_0\omega\sqrt{1-x^2/x_0^2}$

$= \omega\sqrt{x_0^2-x^2}$

$\therefore$ The object executes SHM: (A) $\rightarrow$ (P)

B The graph between v and x :
The object will always move towards x = 0, will slow down in doing so, and will stop at x = 0.



(B) ? ?? (q) , (r)

C. From the elevator frame a pseudo force acts downward. This causes disturbance of equilibrium and the object starts to move downward. But as the extension in the spring increases, the upward force also increases. At some time the object stops momentarily, and then starts moving upward. The object executes SHM.

?C? ? ?? ?p?

D. $v ? 2\sqrt{\frac{GMe}{Re}} ? V_{esc}$

The object escapes from the earth.

?D? ? ?? ?q?

21. Column I gives some devices and **Column II** gives some processes on which the function of these devices depend. Match the devices in **Column I** with the processes in **column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		**Column II**	
(A)	Bimetallic strip	**(p)**	Radiation from a hot body
(B)	Steam engine	**(q)**	Energy conversion
(C)	Incandescent lamp	**(r)**	Melting
(D)	Electric fuse	**(S)**	Thermal expansion of solids

Sol.

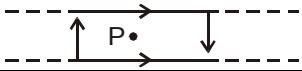
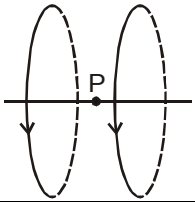
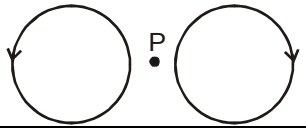
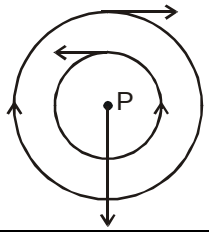
?A? ? ?? ?s?

?B? ? ?? ?q?

?C? ? ?? ?p?, ?q?

?D? ? ?? ?r?, ?q?

22. Two wires each carrying a steady current I are shown in four configurations in **Column I.** Some of the resulting effects are described in **Column II**. Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	Point P is situated Midway between the wires. 	**(p)**	The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(B)	Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	**(q)**	The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C)	Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii. 	**(r)**	There is no magnetic field at P.
(D)	Point P is situated at the common center of the wires. 	**(S)**	The wires repel each other.

Sol.

?A? ? ?? ?q?, ?r?

?B? ? ?? ?p?

?C? ? ?? ?q?, ?r?

?D? ? ?? ?q?,?s?

Chemistry

Part – II

Useful data:

Gas Contant, R = 8.314 K JK^{-1}mol^{-1}
1 F = 96500 C

Atomic Numbers:
H = 1, Li = 3, B= 5, C = 6, N = 7, O = 8, F = 9, Na = 11, P = 15, S = 16, Cl = 17, Ar = 18,
K = 19, V = 23, Cr = 24, Mn = 25, Fe = 26, Co= 27, Ni = 28,Cu = 29, Zn = 30, Ge = 32, Br = 35, Ag = 47,
I = 53, Xe= 54, Pt = 78, Hg = 90 , Pb = 82

Section – I
Straight Objective Type

This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C), and (D), out of which **ONLY ONE** is correct.

23. Consider a reaction aG ? bH ? Products. When concentration of both the reactants G and H is doubled, in the rate increases by eight times. However, when concentration of G is doubled keeping the concentration of H fixed, the rate is doubled. The overall order of the reaction is
(A) 0
(B) 1
(C) 2
(D) 3

Sol. (D)

Rate of reaction = k?G?^{2} ?H?^{1}

24. For the process $H_2O(l)$ (1 bar, 373 K) ? $H_2O(g)$ (1 bar, 373 K), the correct set of thermodynamic parameters is
(A) ?G ? 0, ?S ? ? ve
(B) ?G ? 0, ?S ? ? ve
(C) ?G ? ? ve, ?S ? 0
(D) ?G ? ? ve, ?S ? ? ve

Sol. (A)

?G ? 0, ?S ? ?ve

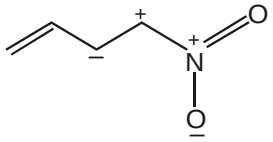
25. A positron is emitted from $^{23}_{11}Na$. The ratio of the atomic mass and atomic number of the resulting nuclide is
(A) 22/10
(B) 22/11
(C) 23/10
(D) 23/12

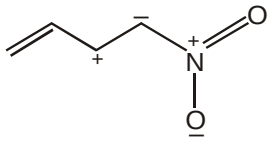
Sol. (C)

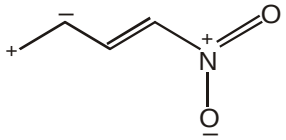
$^{23}_{11}Na \rightarrow {}^{23}_{10}Ne + {}^{0}_{1}e$

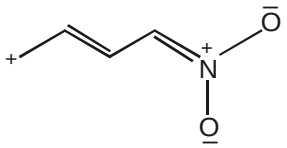
ratio of atomic mass to atomic number $= \frac{23}{10}$

26. Among the following, the least stable resonance structure is

(A) 

(B) 

(C) 

(D) 

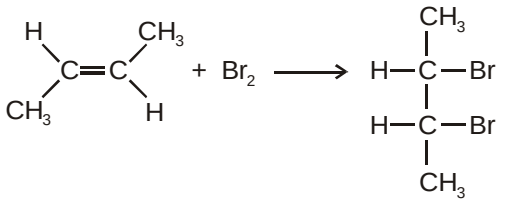
Sol. (A)

Option (A) is least stable because adjacent carbons and nitrogen atoms are having positive charge.

27. The number of stereoisomers obtained by bromination of trans-2 butene is

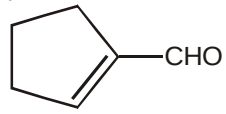
(A) 1
(B) 2
(C) 3
(D) 4

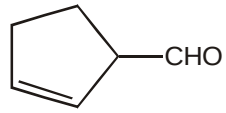
Sol. (A)

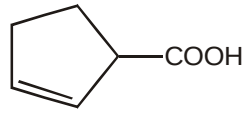


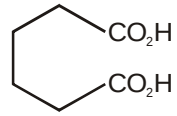
meso compound

28. Cyclohexene on ozonolysis followed by reaction with zinc dust and water gives compound **E.** Compound **E** on further treatment with aqueous KOH yields compound **F.** Compound **F** is

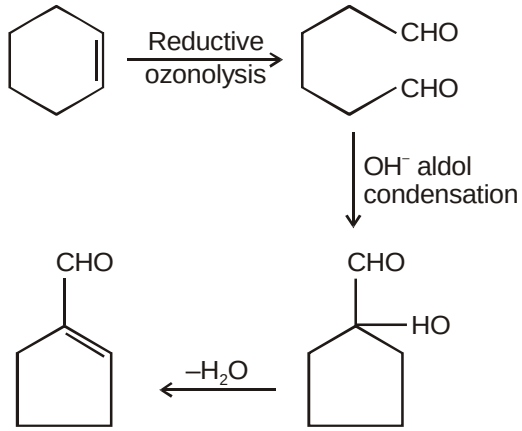
(A) 

(B) 

(C) 

(D) 

Sol. (A)



29. A solution of a metal ion when treated with KI gives a red precipitate which dissolves in excess KI to give a colourless solution. Moreover, the solution of metal ion on treatment with a solution of cobalt(II) thiocyanate give rise to a deep blue crystalline precipitate. The metal ion is

(A) Pb^{2+}

(B) Hg^{2+}

(C) Cu^{2+}

(D) Co^{2+}

Sol. (B)

30. Among the following metal carbonyls, the C — O bond order is lowest in

(A) $[Mn(CO)_6]^+$

(B) $[Fe(CO)_5]$

(C) $[Cr(CO)_6]$

(D) $[V(CO)_6]^-$

Sol. (B)

Back bonding is maximum is $[Fe(CO)_5]$

31. Consider a titration of potassium dichromate solution with acidified Mohr' salt solution using diphenylamine as indicator. The number of moles of Mohr's salt required per mole of dichromate is

(A) 3
(B) 4
(C) 5
(D) 6

Sol. (D)

$Cr_2O_7^{2-} + 6Fe^{2+} + \ldots \rightarrow 2Cr^{3+} + 6Fe^{3+}$

The number of moles of Mohr's salt required per mole of dichromate is six.

Section – II
Assertion – Reason Type

This section contains 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

32. **Statement - 1**
Glucose gives a reddish-brown precipitate with Fehling's solution.

Because

Statement – 2
Reaction of glucose with Fehling's solutions give CuO and gluconic acid.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

Reaction of glucose with Fehlings solution gives Cu_2O and gluconic acid.

33. **Statement - 1**
Alkali metals dissolve in liquid ammonia to give blue solutions.

Because

Statement – 2

Alkali metals in liquid ammonia give solvated species o the type $[M(NH_3)_n]^+$ (M = alkali metals).

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is **Not** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (B)
Blue color appears due to solvated electrons in liquid ammonia.

34. **Statement - 1**
Band gap in germanium is small.

Because

Statement – 2
The energy spread of each germanium atomic energy level is infinitesimally small.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is **NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

35. **Statement - 1**
Molecules that are not superimposable on their mirror images are chiral.

Because

Statement – 2
All chiral molecules have chiral centres.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is **NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

Section – III
Linked Comprehension Type

This section contains 2 paragraphs C36-38 and C39-41. Based upon each paragraph, 3 multiple choice Questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

P36-38 : Paragraph for Questions Nos. 36 to 38.

Redox reactions play a pivotal role in chemistry and biology. The values of standard redox potential (E°) of two half-cell reactions decide which way the reaction is expected to proceed. A simple example is an Daniel cell in which zinc goes into solution and copper gets deposited. Given below are a set of half-cell reactions (acidic medium) along with their E° (V with respect to normal hydrogen electrode) values. Using data obtain the correct explanations to Questions 36-38.

$I_2 + 2e^- \rightarrow 2I^-$	E° = 0.54
$Cl_2 + 2e^- \rightarrow 2Cl^-$	E° = 1.36
$Mn^{3+} + e^- \rightarrow Mn^{2+}$	E° = 1.50
$Fe^{3+} + e^- \rightarrow Fe^{2+}$	E° = 0.77
$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$	E° = 1.23

36. Among the following, identify the correct statement.

(A) Chloride ion is oxidised by O_2

(B) Fe^{2+} is oxidised by iodine

(C) Iodide ion is oxidised by chlorine

(D) Mn^{2+} is oxidised by chlorine

Sol. (C)

Only iodide ion is oxidised by chlorine.

37. While Fe^{3+} is stable, Mn^{3+} is not stable in acid solution because

(A) O_2 oxidises Mn^{2+} to Mn^{3+}

(B) O_2 oxidises both Mn^{2+} to Mn^{3+} and Fe^{2+} to Fe^{3+}

(C) Fe^{3+} oxidises H_2O to O_2

(D) Mn^{3+} oxidises H_2O to O_2

Sol. (D)

38. Sodium fusion extract, obtained from aniline, on treatment with iron(II) sulphate and H_2SO_4 in presence of air gives a Prussian blue precipitate. The blue colour is due to the formation of

(A) $Fe_4[Fe(CN)_6]_3$

(B) $Fe_3[Fe(CN)_6]_2$

(C) $Fe_4[Fe(CN)_6]_2$

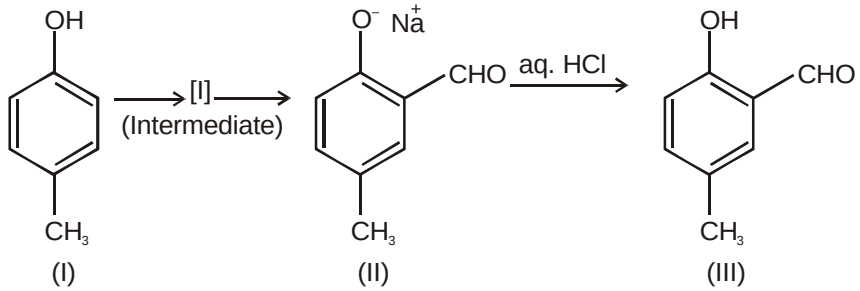
(D) $Fe_3[Fe(CN)_6]_3$

Sol. (A)

Prussian blue is $Fe_4[Fe(CN)_6]_3$

P39-41 : Paragraph for Questions Nos. 39 to 41.

Riemer-Tiemann reaction introduces an aldehyde group, on to the aromatic ring of phenol, ortho to the hydroxyl group. This reaction involves electrophilic aromatic substitution. This is a general method for the synthesis of substituted salicylaldehydes as depicted below.



(I) (II) (III)

39. Which one of the following reagents is used in the above reaction?

(A) aq. NaOH ? CH_3Cl

(B) aq. NaOH ? CH_2Cl_2

(C) aq. NaOH ? $CHCl_3$

(D) aq. NaOH ? CCl_4

Sol. (C)

Reimen-Tiemann reaction takes place is presence of aq. NaOH ? $CHCl_3$

40. The electrophile in this reaction is

(A) :CHCl

(B) ? $CHCl_2$

(C) : CCl_2

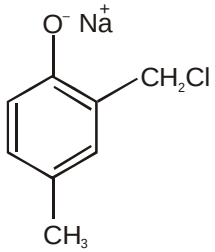
(D) : CCl_3

Sol. (C)

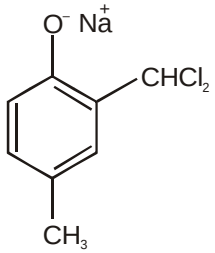
It is dichloro carbon.

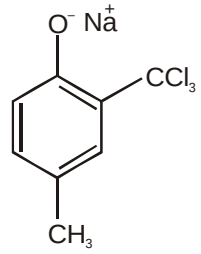
41. The structure of the intermediate **I** is

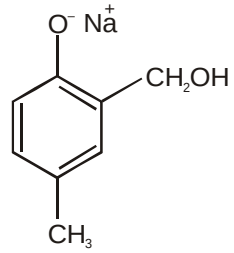
(A)



(B)



(C) 

(D) 

Sol. (B)

Section – IV
Matrix-Match Type

This section contains 3 questions . Each question contains statement given in two columns which have to be matched. Statement (A, B, C, D) in **column I** have to be matched with statements (p, q, r, s) in **column II.** The answers to these questions have to be appropriately bubbled as illustrated in the following examples.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s then the correct bubbled 4 × 4 matrix should be as follows

	p	q	r	s
A	Ⓟ	Ⓠ	Ⓡ	Ⓢ
B	Ⓟ	Ⓠ	Ⓡ	Ⓢ
C	Ⓟ	Ⓠ	Ⓡ	Ⓢ
D	Ⓟ	Ⓠ	Ⓡ	Ⓢ

42. Match the complexes in **column I** with nature of the reactions/type of the products in **column II.** Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	$O_2^?$? O_2 ? $O_2^{2?}$	**(p)**	redox reaction
(B)	$CrO_4^{2?}$? H^4 ?	**(q)**	one of the products has trigonal-planar structure
(C)	$MnO_4^?$? $NO_2^?$? $H^?$?	**(r)**	dimeric bridged tetrahedral metal ion
(D)	$NO_3^?$? H_2SO_4 ? $Fe^{2?}$?	**(s)**	disproportionation

Sol.

A – p, s
B – r
C – p, q
D – p

43. Match the chemical system/units cells mentioned in **column I** with their characteristic features mentioned in **column II.** Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	simple cubic and face-centred cubic	**(p)**	have these cell parameters a = b = c and ? ? ? ? ?
(B)	cubic and rhombohedral	**(q)**	are two crystal systems
(C)	cubic an tetragonal	**(r)**	Have only tow crystallographic angles of 90°
(D)	hexagonal and monoaclinic	**(s)**	Belong to same crystal system

Sol.

A – p s
B – p, q
C – q
D – r, q

44. Match the compounds/ions in **Column I** with their properties/reactions in **Column II.** Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	C_6H_5CH0	**(p)**	gives precipitate with 2, 4-dinitrophenylhydrazine
(B)	CH_3 C ? CH	**(q)**	gives precipitate with $AgNO_3$
(C)	$CN^?$	**(r)**	is a nucleophile
(D)	$I^?$	**(s)**	is involved in cyanohydrin formation

Sol.

A – p, q, s
B – q
C – q, r, s
D – q, r

Mathematics

Section – III
Straight Objective Type

This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct

45. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of worlds that appear before the word **COCHIN** is
(A) 360
(B) 192
(C 96
(D) 48

Sol. (C)

CC ... 4!

CH ... 4!

CI ... 4!

CN ... 4!

C O C H I N

4!? 4 ? 96 words before COCHIN

46. Let $f(x) ? \frac{x}{?1 ? x^n ?^{1/n}}$ for $n ? 2$ and $g(x) ? ?\underbrace{f \; f \; ??? \; f}_{\text{f occurs n times}}?(x)$. Then $?x^{n?2}\, g(x)\, dx$ equals

(A) $\frac{1}{n ?n ? 1?} ?1 ? nx^n ?^{1?\frac{1}{n}} ? K$

(B) $\frac{1}{n ? 1} ?1 ? nx^n ?^{1?\frac{1}{n}} ? K$

(C) $\frac{1}{n ?n ? 1?} ?1 ? nx^n ?^{1?\frac{1}{n}} ? K$

(D) $\frac{1}{n ? 1} ?1 ? nx^n ?^{1?\frac{1}{n}} ? K$

Sol. (A)

Let n = 2

$f(x) ? \frac{x}{\sqrt{1 ? x^2}}$

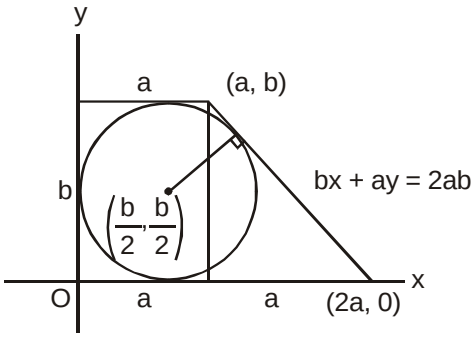
$g(x) ? f ?f ?x?? ? \frac{x}{\sqrt{1 ? 2x^2}}$

$?g ?x ?dx ? \frac{\sqrt{1 ? 2x^2}}{2} ? C$

47. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2 CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(A) 3
(B) 2
(C) $\frac{3}{2}$
(D) 1

Sol. (B)



$$\left|\frac{\frac{b^2}{2}+\frac{ab}{2}-2ab}{\sqrt{a^2+b^2}}\right| = \frac{b}{2}$$

$$|b-3a| = \sqrt{a^2+b^2}$$

$$9a^2+b^2-6ab = a^2+b^2$$

$$8a^2 = 6ab$$

$$4a = 3b$$

also, $ab - \frac{1}{2}ab - \frac{3}{2}ab = 18$

$\Rightarrow ab = 12$

$a = 3, b = 4$

radius = 2

48. Let $a, b, c,$ be unit vectors such that $a + b + c = 0$. Which one of the following is correct?

(A) $a \cdot b + b \cdot c + c \cdot a = 0$

(B) $a \cdot b + b \cdot c + c \cdot a = 0$

(C) $a \cdot b + b \cdot c + a \cdot c = 0$

(D) $a \cdot b,\ b \cdot c,\ c \cdot a$ are mutually perpendicular

Sol. (B)

$a \cdot b + b \cdot c + c \cdot a = 0$

$a + b + c = 0$

$a + b = -c$

$c \cdot a + c \cdot b + 0 = c \cdot a + b \cdot c$

Similarly $a \cdot b + b \cdot c + c \cdot a = 0$ as a, b and c are vectors along equilateral Δ as $|a| = |b| = |c| = 1$

49. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at (0, 1)

(B) variable radii and a fixed centre at $(0, -1)$

(C) fixed radius 1 and variable centres along the x-axis

(D) fixed radius 1 and variable centres along the y-axis

Sol. (C)

$$\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{y}$$

$$\frac{ydy}{\sqrt{1 - y^2}} = dx$$

$$-\frac{1}{2}\int \frac{-2ydy}{\sqrt{1 - y^2}} = \int dx$$

$$-\frac{1}{2} \cdot 2\sqrt{1 - y^2} = x + c$$

$$-\sqrt{1 - y^2} = x + c$$

Squaring we get

$1 - y^2 = x^2 + 2cx + c^2$

$x^2 + y^2 + 2cx + c^2 - 1 = 0$

Centre (–c, 0); radius = 1

50. $\frac{d^2x}{dy^2}$ equals

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

(B) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

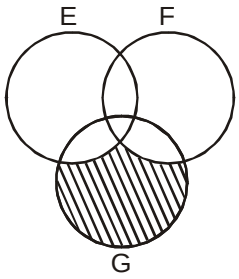
Sol. (D)

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{\frac{d}{dx}\left(\frac{dx}{dy}\right)}{\frac{dy}{dx}} = \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}}{\frac{dy}{dx}}$$

$$= \frac{\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-3}$$

51. Let E^c denote the complement of an event of an event E. Let E, F, G be pariwise independent events with P (G) > 0 and $P(E^C \cap F \cap G) = 0$. Then $P(E^C \cap F^C \mid G)$ equals

Sol. (C)



52. if $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on

(A) a line not passing through the origin

(B) $|z| = \sqrt{2}$

(C) the x-axis

(D) the y-axis

Sol. (D)

$|z| = 1$ let $z = \cos\theta + i\sin\theta$

$$\frac{z}{1 - z^2} = \frac{\cos\theta + i\sin\theta}{1 - \cos 2\theta - i\sin 2\theta}$$

$$= \frac{\cos\theta + i\sin\theta}{2\sin^2\theta - 2i\sin\theta\cos\theta}$$

$$= \frac{\cos\theta + i\sin\theta}{-2i\sin^2\theta - i(\cos\theta + i\sin\theta)}$$

$$= \frac{i}{2\sin\theta} = \text{purely imaginery}$$

$\Rightarrow \frac{z}{1 - z^2}$ lie on y-axis.

53. Let O (0, 0), P (3, 4), Q (6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

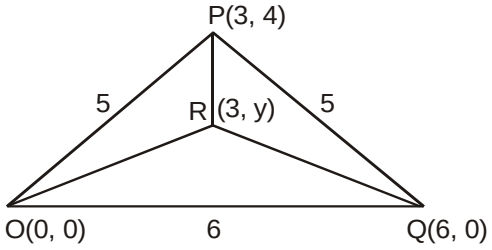
(A) $\left(\frac{4}{3}, 3\right)$

(B) $\left(3, \frac{2}{3}\right)$

(C) $\left(3, \frac{4}{3}\right)$

(D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

Sol. (C)



$\Delta PQR = \frac{1}{2} \times 6 \times 4 = 12$ sq. units

$\Delta ROQ = \frac{1}{2} \times y \times 6 = 3y$

$3y = 4$

$y = \frac{4}{3}$

Section – II
Assertion – Reason Type

This section contains 4 question numbered 54 to 57. Each question contains STATEMENT 1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

54. **Statement-1:** The curve $y = \frac{-x^2}{2} + x - 1$ is symmetric with respect to the line x = 1.

Because

Statement – 2
A parabola is symmetric about its axis.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is **NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (A)

$$y = \frac{x^2}{2} - x - 1$$

$$\frac{dy}{dx} = x - 1 = 0 \Rightarrow x = 1$$

Clearly $y = ax^2 + bx + c$ is symmetric about $x = \frac{-b}{2a}$ i.e. its axis.

55. Consider the planes $3x - 6y - 2z = 15$ and $3x + y + 2z = 5$.

Statement-1: The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$.

Because

Statement-2: The vector $14\hat{i} + 2\hat{j} - 15\hat{k}$ is parallel to the line of intersection of given planes.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is **NOT** a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (D)

$$3x + 6y = 15 + 2z$$

$$2x + y = 5 + 2z$$

$$15x = 45 + 14z \Rightarrow x = \frac{45 + 14z}{15}, \quad y = \frac{2z - 15}{15}$$

If $z = 15t$
$x = 3 + 14t$
$y = 2t - 1$

56. Lines $L_1: y - x = 0$ and $L_2: 2x + y = 0$ intersect the line $L_3: y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement - 1
The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

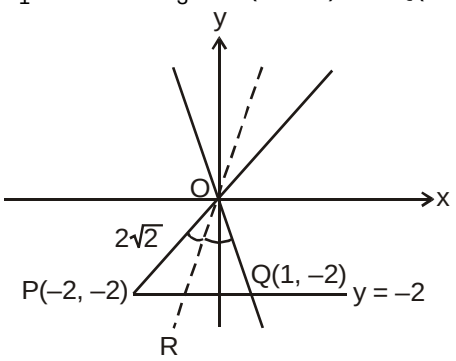
Because

Statement – 2
In any triangle, bisector of an angle divides the triangle into two similar triangles.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False
(D) Statement – 1 is False, Statement – 2 is True

Sol. (C)

Statement 2 is false

L_1 intersects L_3 at P(–2, –2) and Q(1, –2)



$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$

57. Let $f(x) = 2 + \cos x$ for all real x.

Statement - 1

For each real t, there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$.

Because

Statement – 2

$f(t) = f(t + 2\pi)$ for each real t.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for statement – 1

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is Not a correct explanation for Statement – 1.

(C) Statement – 1 is True, Statement – 2 is False

(D) Statement – 1 is False, Statement – 2 is True

Sol. (B)

In only interval $[t, t + \pi]$ $\exists$ c such that $f'(c) = 0$ and also $f(d) = f[t + 2\pi]$

but statement 2 doesn't explain statement 1.

M58-60 : Paragraph for Questions Nos. 58 to 60

Let A_1, G_1, H_1 denoted the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

58. Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > ...$

(B) $G_1 < G_2 < G_3 < ...$

(C) $G_1 = G_2 = G_3 = ...$

(D) $G_1 = G_3 = G_5 = ...$ and $G_2 = G_4 = G_6 = ...$

Sol. (C)

$A_1 = \frac{a + b}{2}$

$G_1 = \sqrt{ab}$

$H_1 = \frac{2ab}{a + b}$

$A_n = \frac{A_{n-1} + H_{n-1}}{2}$

$G_n = \sqrt{A_{n-1} H_{n-1}}$

$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$

$G_2 = \sqrt{A_1 H_1} = \sqrt{ab} = G_1$

$G_3 = \sqrt{A_2 H_2} = \sqrt{\frac{A_1 + H_1}{2} \cdot \frac{2A_1H_1}{A_1 + H_1}} = G_1$

$= G_1 = G_2 = G_3 = ...$

59. Which one of the following statements is correct?

(A) $A_1 < A_2 < A_3 < ...$

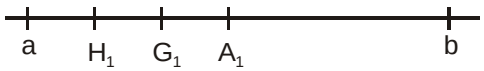
(B) $A_1 > A_2 > A_3 > ...$

(C) $A_1 < A_3 < A_5 < ...$ and $A_2 > A_4 > A_6 > ...$

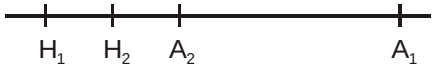
(D) $A_1 > A_3 > A_5 > ...$ and $A_2 < A_4 < A_6 < ...$

Sol. (A)

assume a < b



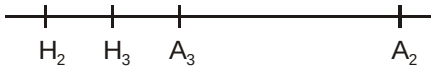
$A_2 = \frac{A_1 + H_1}{2}, H_2 = \frac{2A_1H_1}{A_1 + H_1}$



$= A_2 < A_1$

Similarly

$A_3 = \frac{A_2 + H_2}{2}, H_3 = \frac{2A_2H_2}{A_2 + H_2}$



$= A_3 < A_2$

$A_1 < A_2 < A_3 < ....$

60. Which one of the following statements is correct?

(A) $H_1 < H_2 < H_3 < ...$

(B) $H_1 > H_2 > H_3 > ...$

(C) $H_1 < H_3 < H_5 < ...$ and $H_2 > H_4 > H_6 > ...$

(D) $H_1 > H_3 > H_5 > ...$ and $H_2 < H_4 < H_6 < ...$

Sol. (B)

As shown above

$H_1 ? H_2 ? H_3 ? ....$

M61-63 : Paragraph for Questions Nos. 63-63

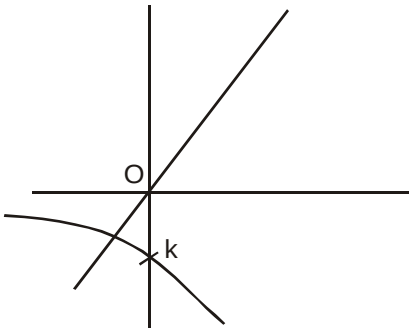
If a continuous function f defined on the real line R, assumes positive and negative values in R then the equation f(x) = 0 has a root in r. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative then the equation f(x) = 0 has a root in R.

Consider $f(x) ? ke^x ? x$ for all real x and k is a real constant.

61. The line y = x meets $y ? ke^x$ for $k ? 0$ at

(A) no point
(B) one point
(C) two points
(D) more than two points

Sol. (B)



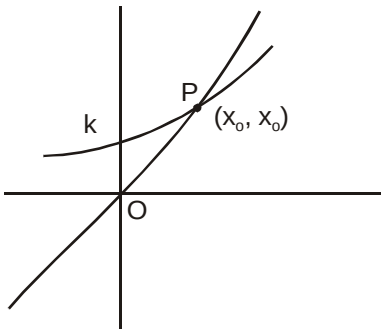
$x ? ke^x, k ? 0$

even if k = 0 i.e. x-axis cuts y = x at one point.

62. The positive value of k for which $ke^x ? x ? 0$ has only one root is

(A) $\frac{1}{e}$
(B) 1
(C) e
(D) $\log_e 2$

Sol. (A)



$k > 0$

$x_0 = ke^{x_0}$ and $\left(\frac{dy}{dx}\right)_P = 1 = ke^{x_0} = 1$

$\Rightarrow x_0 = 1$

$\Rightarrow k = \frac{1}{e}$

63. For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is

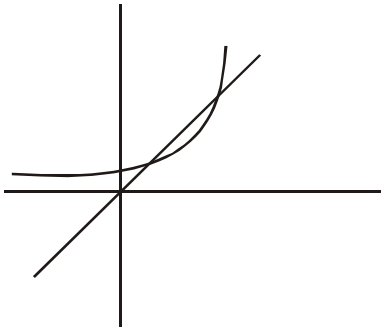
(A) $\left(0, \frac{1}{e}\right)$

(B) $\left(\frac{1}{e}, 1\right)$

(C) $\left(\frac{1}{e}, \infty\right)$

(D) (0, 1)

Sol. (A)



$k > 0$

Similarly as above $0 < k < \frac{1}{e}$

Section – IV
Matrix-Match Type

This section contains 3 questions . Each question contains statement given in two columns which have to be matched. Statement (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following examples.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s then the correct bubbled 4 × 4 matrix should be as

Follows

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

64. Let $f(x) ? \frac{x^2 ? 6x ? 5}{x^2 ? 5x ? 6}$

Match the expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

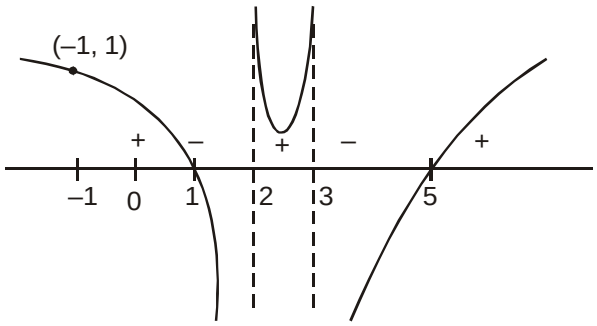
Column I		Column II	
(A)	If –1 < x < 1, then f(x) satisfies	**(p)**	0 < f(x) < 1
(B)	If 1 < x < 2, then f(x) satisfies	**(q)**	f(x) < 0
(C)	If 3 < x 5, then f(x) satisfies	**(r)**	f(x) > 0
(D)	If x > 5, then f(x) satisfies	**(S)**	f(x) < 1

Sol.

A ? p, r, s; B ? q, s; C ? q, s; D ? p, r, s

$f(x) ? \frac{x^2 ? 6x ? 5}{x^2 ? 5x ? 6}$

$? \frac{?x ? 5??x ? 1?}{?x ? 3??x ? 2?}$



In –1 < x < 1, f(x) > 0 and f(x) < 1

A ? p, r, s

If 1 < x < 2

f(x)< 0

? B ? q , s

If 3 < x < 5, f(x) < 0

? C ? q , s

If x > 5, f(x) > 0

D ? p, r, s

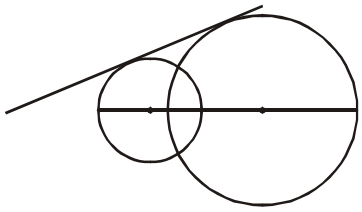
65. Match the statements in Column I with the properties in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	Two intersecting circles	**(p)**	have a common tangent
(B)	Two mutually external circles	**(q)**	have a common normal
(C)	Two circles, one strictly inside the other	**(r)**	do not have a common tangent
(D)	Two branches of a hyperbola	**(S)**	do not have a common normal

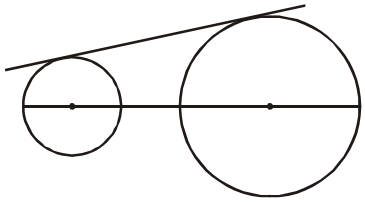
Sol.

A ? p, q, B ? p, q, C ? r, q, D ? r, q

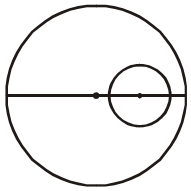
A ? p, q



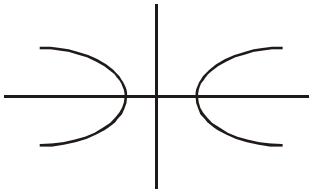
B ? p, q



C ? q, r



D ? r, q



66. Let (x, y) be such that

$\sin^{?1}(ax) ? \cos^{?1}(y) ? \cos^{?1}(bxy) ? \frac{?}{2}$

Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	If a = 1 and b = 0, then (x, y)	**(p)**	lies on the circle $x^2 ? y^2 ? 1$
(B)	If a = 1 and b = 1, then (x, y)	**(q)**	lies on $(x^2 ? 1)(y^2 ? 1) ? 0$
(C)	If a = 1 and b = 2, then (x, y)	**(r)**	lies on y = x
(D)	If a = 2 and b = 2, then (x, y)	**(S)**	lies on $(4x^2 ? 1)(y^2 ? 1) ? 0$

Sol.

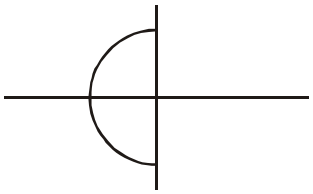
A ? p, B ? q, C ? p, D ? r
a = 1, b = 0

$\Rightarrow \sin^{-1}x + \cos^{-1}y = 0$

$\Rightarrow \cos^{-1}y = \sin^{-1}(-x)$

$\Rightarrow \sin(\cos^{-1}y) = -x$

$\Rightarrow \sqrt{1-y^2} = -x$ or $x = -\sqrt{1-y^2}$



B $\rightarrow$ q

a = 1, b = 1

$\Rightarrow \sin^{-1}x + \cos^{-1}y + \cos^{-1}(xy) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}y + \cos^{-1}xy = \frac{\pi}{2} - \sin^{-1}x = \cos^{-1}x$

$\Rightarrow \cos^{-1}y + \cos^{-1}xy = \cos^{-1}x$

Take cos both sides

$\Rightarrow y.xy - \sqrt{1-y^2}\sqrt{1-x^2y^2} = x$

$\Rightarrow x(y^2-1) = -\sqrt{1-y^2}\sqrt{1-x^2y^2}$

$\Rightarrow -x\sqrt{1-y^2} = \sqrt{1-x^2y^2}$ or $\sqrt{1-y^2} = 0$

$\Rightarrow x^2(1-y^2) = 1-x^2y^2$

$\Rightarrow x^2 = 1$ or $y^2 = 1$

i.e. $(x^2-1)(y^2-1) = 0$

a = 1, b = 2

$\sin^{-1}x + \cos^{-1}y + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\cos^{-1}y + \cos^{-1}(2xy) = \cos^{-1}x$

$2xy^2 - \sqrt{1-y^2}\sqrt{1-x^2y^2} = x$

$x(2y^2-1) = \sqrt{1-y^2}\sqrt{1-4x^2y^2}$

Squaring we get

$x^2(2y^2-1)^2 = (1-y^2)(1-4x^2y^2)$

$x^2(4y^4 - 4y^2 + 1) = 1-4x^2y^2 - y^2 + 4x^2y^4$

$4x^2y^4 - 4x^2y^2 + x^2 = 1-4x^2y^2 - y^2 + 4x^2y^4$

$x^2 + y^2 = 1$

C $\rightarrow$ p

a = 2; b = 2

$\sin^{-1}2x + \cos^{-1}y + \cos^{-1}2xy = \frac{\pi}{2}$

$\cos^{-1}y + \cos^{-1}\left(2xy\sqrt{1-y^2}\sqrt{1-4x^2y^2}\right) = \cos^{-1}2x$

$y\sqrt{1-4x^2y^2} + 2xy\sqrt{1-y^2} = 2x$

$2x\left(y^2 + 1\sqrt{1-y^2}\sqrt{1-4x^2y^2}\right)$

$y^2 = 1$ or $2x\sqrt{1-y^2} = \sqrt{1-4x^2y^2}$

$4x^2\left(1-y^2\right) = 1-4x^2y^2$

$4x^2 = 1$

D = r